Cashflow at Risk (CFaR) for Tax-Exempt Liability Management

Intuitive Analytics

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"In the 21st Century, scientists and engineers will continue to inform us regarding what we can do with our ever-expanding knowledge base, but it is our shared responsibility to decide what we should do. And deciding what we should do is the greatest responsibility we all bear as we move forward together."

Harold T. Shapiro

Overview

"My head's in the freezer and my feet are in the oven but on average, I feel fine."
Anonymous

Many analyses prepared for decision makers today suffer a very serious shortcoming, particularly when it comes to decisions related to market risk. Frequently, the only meaningful quantitative information processed is based upon a single historic or expected average of the market element(s) in question – an interest rate, investment growth rate, commodity price, etc. In the tax-exempt capital markets, this often consists of using a single historic average for tax-exempt short term rates, taxable short term rates, and/or the ratio between the two. The resulting elephant in the living room is, of course, that these assumptions are wrong. And everyone knows they're wrong. The BMA index will not be 3.50% for the next 30 years. LIBOR will not be 6% for the next 20 years. The crucial question beyond, "But what can we do about it?" is "How wrong are these assumptions?" This article, in describing Cash Flow at Risk (CFaR), tries to at least partially address both of those questions.

For many years, finance professionals analyzing debt have measured risk by dividing the aggregate amount of outstanding principal deemed adjustable rate or "floating" by total debt outstanding. The resulting ratio is often called a "fixed/floating mix" or simply a "debt mix." Though an admittedly simple measure to calculate and understand, this benefit is counterbalanced by major shortcomings. These shortcomings are even more pronounced given the size and complexity of the debt and derivative structures that are now commonplace.

This article details a methodology for calculating CFaR as an alternative to fixed/floating mix, explains CFaR's intuitive meaning graphically, and using CFaR compares different financial instrument portfolios through changing market expectations and model assumptions.

Value at Risk – the Origin of CFaR

The idea behind CFaR is rooted in a risk management concept developed in the 1980s called Value at Risk (VaR). VaR is used by investment portfolio managers, banks, insurance companies, regulators, and others in assessing the probability and magnitude of *mark-to-market* loss in a portfolio of market sensitive instruments.[1] Specifically, VaR is the minimum expected loss at a specified confidence interval, usually 95% or 99%, over a given time horizon. If the mention of "confidence interval" has introduced a cerebral fog not experienced since college statistics (sometimes more aptly called "sadistics"), fear not, VaR has a visual representation that will carry over in a straightforward manner to the explanation of CFaR.

[1]Banks are required to report 14 day VaR to regulators and are subject to VaR driven capital requirements.

Figure 1

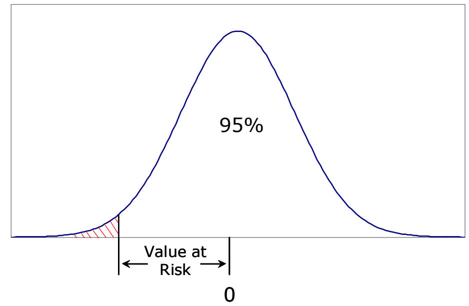


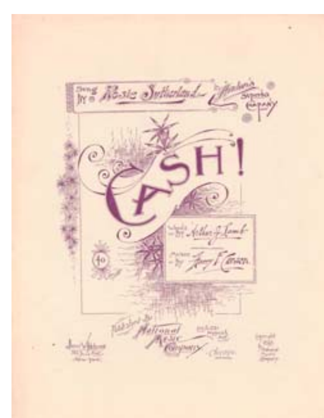
Figure 1 represents the probability distribution of possible returns from an investment portfolio. Note that the "expected" return is slightly above 0, i.e. the peak of the bell curve is just to the right of the 0 point on the horizontal axis. However, the VaR is defined as the minimum expected loss (amount below 0) that might occur at a specified confidence interval. Put differently, if we're calculating a 95% VaR we need to look at the distribution of modeled returns (or price changes) and go to the point at which only 5% of the distribution is lower: in the graph [see Figure 1] this is reflected by the vertical line on the right border of the red area. The amount by which that point is negative is the 95% VaR.

At this point it's important to note that the calculation above requires that there exist some underlying return model that describes not only how the value of the portfolio is *expected* to change, but also the variability of the portfolio value about that expectation.

Translating VaR to Cash Flow at Risk

VaR is a valuable tool for understanding market to market risk which is relevant for banks, insurance companies, and others with large trading portfolios. However, both public and private corporations with debt portfolios are often more concerned with ongoing budget and interest expense variability than the market value changes in their liabilities. This is more appropriately addressed through Cash flow at Risk (CFaR).

CFaR has a similar meaning to VaR, though with some key differences. Because a debt portfolio of bonds, by definition, must already have an associated cost (debt capital is not free), we must measure CFaR relative to something other than zero. Unlike VaR, CFaR is not an absolute dollar number indicating what might be lost in value over a particular horizon. Debt service will have some expected cost out of the gate, and it is *against that expectation* that CFaR is measured. The other important difference is that tax-exempt debt service cost usually spans many years, so there will be many different CFaRs. For example, there may be one for each annual budget period. How should a single number be determined from the many numbers generated in a complete CFaR analysis? Average them? Take the maximum? In this article, we define CFaR as the maximum or peak CFaR across the debt service horizon; taking the maximum seems only prudent given this is a measure of risk after all. Cash Flow at Risk in this article is defined as follows:



> **Cash Flow at Risk (CFaR)** – The maximum increase in cost, relative to a particular expectation, that could be experienced due to the impact of market risk on a specified set of financial instruments (bonds, swaps, investments, etc.), over a particular period of time and selected confidence level.

In the next section, the meaning of this statement, together with an intuitive graphical representation and a calculation methodology, are offered.

Measuring CFaR

“A reasonable probability is the only certainty”
Edward Watson Howe

In order to calculate either VaR or CFaR we must employ some sort of market model that meaningfully describes the dynamics of relevant market variables (interest rates, equity prices, commodities, currencies, etc). For CFaR related to a liability portfolio, the model must provide a distribution for interest rates which will in turn lead to a distribution of cash flows through time.

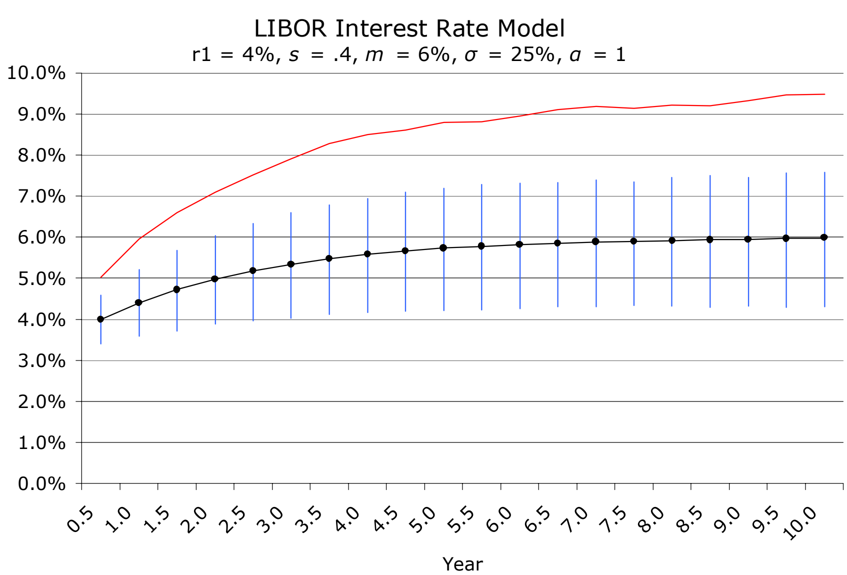
In the tax-exempt market, it is generally safe to assume that three market rates will drive the majority of cashflow variability and CFaR arising from an issuer’s debt portfolio: tax-exempt short term rates often proxied by the BMA index; taxable short term rates such as LIBOR; and very importantly as we’ll see, the relationship between the two often called the “BMA/LIBOR ratio.” An analysis leading to a comprehensive CFaR number for a tax-exempt issuer must include a model of at least these three market elements.[2]

[2] With constant innovation leading to ever more creative structures, and in particular, the recent and widespread popularity of constant maturity swaps (CMS), this may soon include a fourth element: term CMS rates.

Though some may consider it a necessary evil, calculating CFaR will require the use of one or more interest rate models. Interest rate models are often uttered only in hushed terms and frequently in conjunction with things like “rocket science” and “Wall Street quants”. The fact is that anyone who’s heard of a bell curve and can intuitively grasp the concept that interest rates in the future are more likely to be in certain ranges than others is fully capable of having a very profitable understanding of interest rate models. A complete description of a quite powerful yet simple interest rate model, fully functional with a grand total of four inputs, is provided elsewhere.[3]

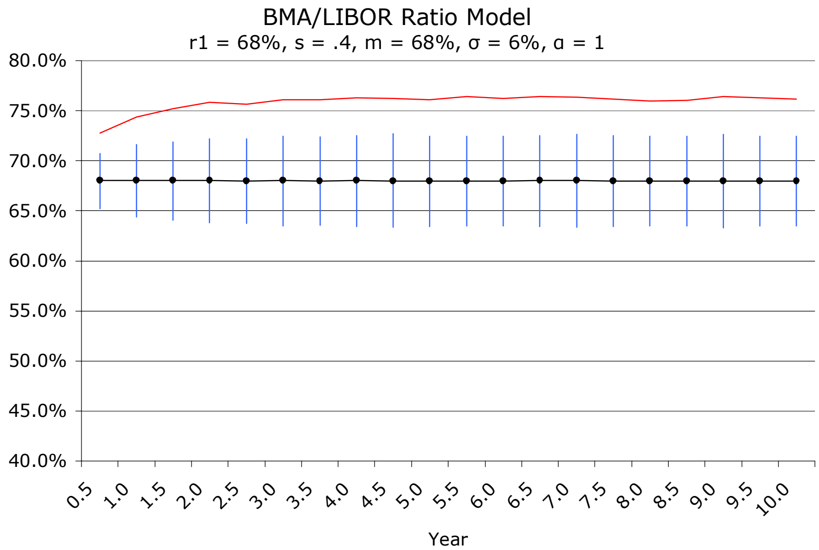
[3] “The Time has Come: Interest Rate Modeling for Liability Management” and companion spreadsheet.

The graph below depicts the output from an interest rate model for LIBOR.

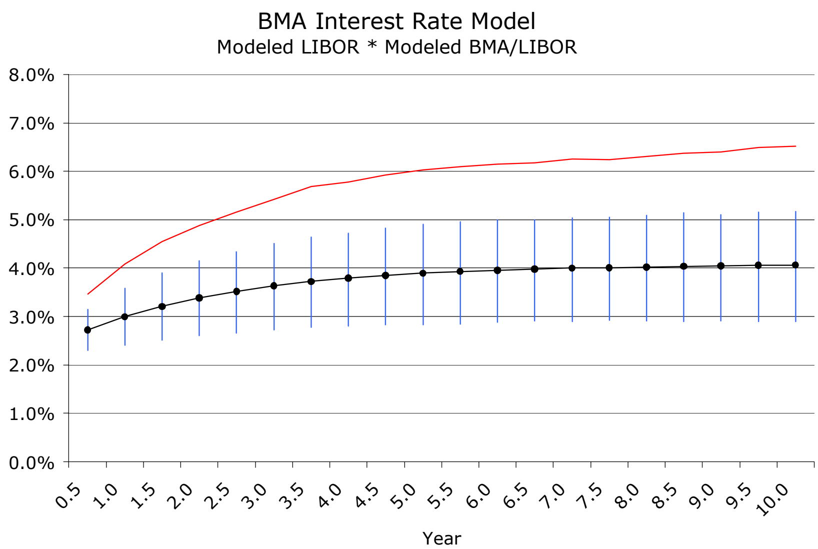


Without going into great detail, the average rate in this short term LIBOR rate model starts at 4% and grows to 6% over time. The blue bars represent plus and minus one standard deviation, and the red line shows the 95% highest rate at each semi-annual time step over the ten year analytic horizon.

Using different input parameters but the same model, we can easily generate BMA/LIBOR ratios themselves. In this case, modeled ratios start at 68% and remain there, on average through the analytic horizon. Importantly, however, we've also captured the fact that there is volatility in this relationship.



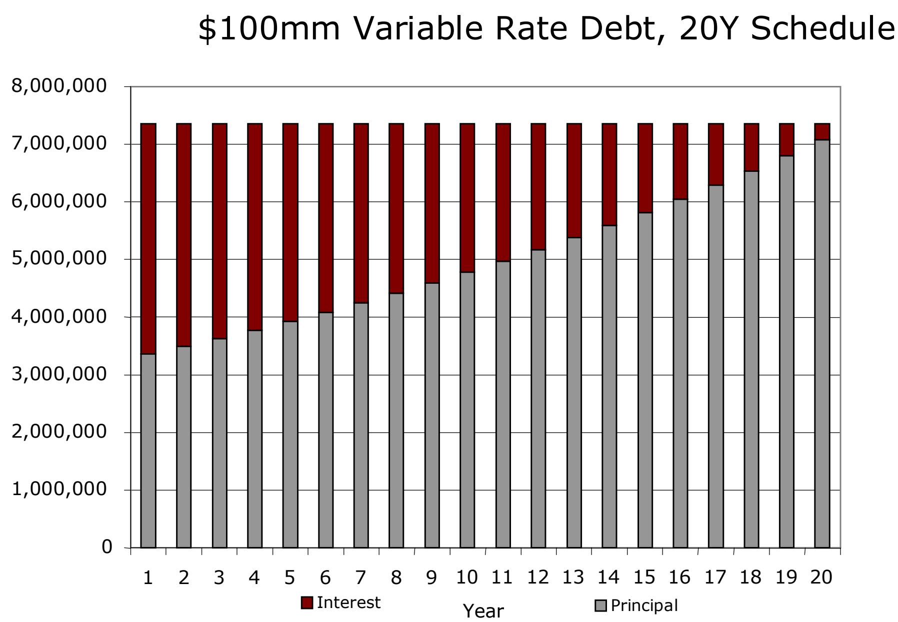
Now suitably armed with models for the three underlying market rates that will drive cashflow variability in the tax-exempt market, in the next two sections we'll explore six sample issuer portfolios, their respective CFaRs, and the impact of changing markets and model parameters.



Examples and Interpretation of CFaR

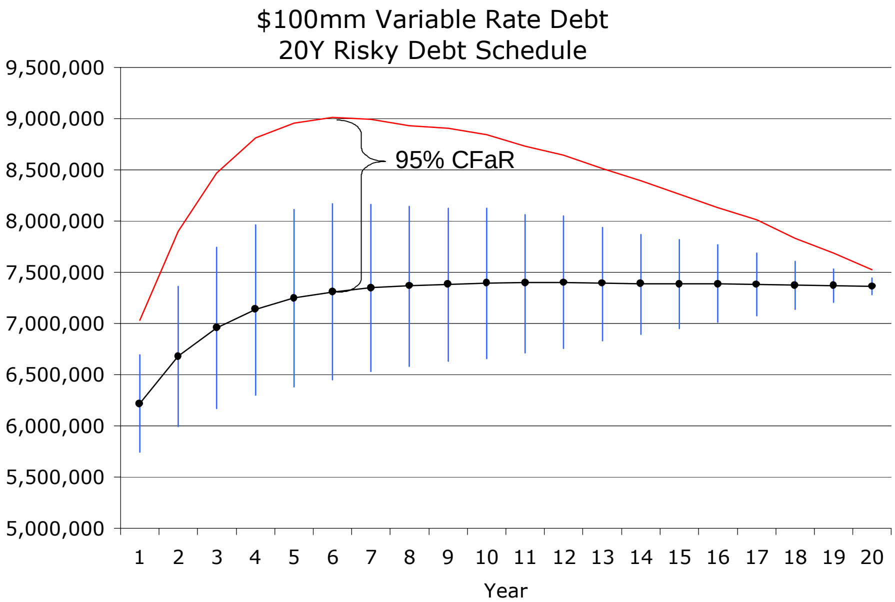
"Never try to walk across a river just because it has an average depth of three feet."
Milton Friedman

In this section, we'll introduce the concept of cash flow variability and ultimately, calculate CFaR. To do that we'll start with a simple, hypothetical example of a 20 year, $100 million variable rate bond issue (VRBs) with a level debt service structure assuming a 4% floating rate. For this structure, a traditional debt service schedule is usually illustrated by a bar chart with interest amounts reflected by one color, below in red, and principal as another, below in grey.



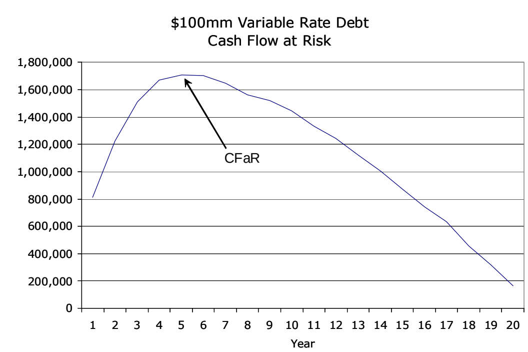
Annual debt service at a 4% interest rate is approximately $7.3 million and as we expect, interest amounts decline as principal is retired. This is informative in that it offers some sense of the relative magnitudes of principal and interest as debt is retired but it offers little if any insight into how high or low total debt service payments might go given interest rate movements. For this information, we must deploy the BMA interest rate model described in the prior section.

As an alternative to the traditional bar chart above, below is a graphic of the same debt service schedule though incorporating the reasonable and even important assumption that variable rates, well...vary. The statistical information in this chart is identical to that shown in the charts above for interest rates, though this chart illuminates an essential piece of information: the degree to which interest rates could potentially influence the magnitude of annual debt service. The expected debt service payment is reflected by the black circles with plus and minus one standard deviation confidence intervals reflected in blue. Here we see that, given the BMA interest rate model above, expected debt service is **not** actually level. Modeled BMA rates start at approximately 2.90 and rises to approximately 4% over the period.[4]



This might be considered a "risky debt schedule" as it visually reveals the market risk influence on a capital structure's cash flows. Though our uncertainty about interest rates (and the "width" of the interest rate distribution) increases with time, we see in this schedule the countervailing effect of retiring principal. It is this effect that results in the shrinking width of the blue lines as the issuer approaches maturity; interest expense is an ever smaller component of each debt service payment.

The graph at right shows the annual difference between the expected debt service (the black line above) and the 95% worst case (the red line above). Here it's clear; we see that annual CFaR peaks in year five at $1.71 million. Therefore, $1.71 million is the 95% CFaR for this pure variable rate liability structure.



This would all be pointless if we weren't able to apply this methodology to the range of instruments currently comprising modern capital structure profiles. To address the trivial case first, as CFaR has been defined, a pure fixed rate bond series or portfolio exhibits a CFaR of zero, given its nominally constant cash flow. However, we'd expect such a portfolio to also exhibit the highest expected cost. Note again the difference here between the perspective of investor and issuer: an investor would consider a fixed rate bond, particularly one with any significant duration, as a risky instrument.

Departing from BOBs (affectionately, Boring Old Bonds) we now apply the same methodology to five other portfolios. Starting with the variable rate bonds already described, the complete list of portfolios is as follows:

[4] A natural question arising from this construction is whether or not the original principal amortization should be adjusted given our expectation for BMA is not a constant 4%. However, the proper treatment of that question deserves its own paper.

Portfolio 1 – $100mm Variable Rate Bonds (VRBs)
Portfolio 2 – $100mm VRBs and $20mm cash
Portfolio 3 – $100mm VRBs, $100mm 68% LIBOR swap to fixed
Portfolio 4 – $100mm VRBs , $100mm BMA/LIBOR basis swap
Portfolio 5 – $200mm VRBs, $100mm 68% LIB and basis swaps
Portfolio 6 – Portfolio 5 plus $20 mm cash

The CFaR analysis of each of these portfolios is provided in the remainder of this section.

Portfolio 1 – $100mm VRBs
Portfolio 1 consists solely of our $100 million floating rate bond issue. Its CFaR is $1.71 million reflecting all the risk in the variable of BMA rates as modeled. Only Portfolio 5 with its $200 million in floaters and swaps has greater CFaR.

Portfolio 2 – $100mm VRBs and $20mm cash
Cash is modeled assuming that it earns the prevailing LIBOR rate. Though money market fund earnings have not historically tracked LIBOR rates exactly, they are highly correlated leading to the assumption here. In fact, money market fund earnings are likely to earn some positive spread to LIBOR over time.

As would be expected, the CFaR of the VRBs and cash is lower than the VRBs alone in Portfolio 1. CFaR in Portfolio 2 is $1.24 million representing a 27% reduction in CFaR from Portfolio 1. This is intuitively satisfying given we expect a $1 of LIBOR based investments to hedge more than a $1 of tax-exempt VRBs given the tax-exempt/taxable yield ratio. While we're here, it is important to note that this is materially *less* reduction in risk than expected based upon the average 68% BMA/LIBOR ratio in the model. This perhaps surprising result derives from the fact that the BMA rate and LIBOR rate are not 100% correlated; there is a certain amount of "noise" between the two which is consistent with empirical evidence and likely a reasonable assumption going forward.

Portfolio 3 – $100mm VRBs, $100mm 68% LIBOR swap to fixed
Portfolio 3 adds to the VRBs in Portfolio 1 a 68% LIBOR swap where the issuer pays 4% and receives 68% LIBOR. We would clearly expect to see a reduction in CFaR given the swap and this is in fact the case: CFaR is $390,000 for Portfolio 3, a 77% reduction relative to CFaR in Portfolio 1.

Portfolio 4 – $100mm VRBs, $100mm 78% BMA/LIB Basis Swap
Portfolio 4 adds to the VRBs a 78% BMA/LIBOR basis swap where the issuer pays the BMA index and 78% LIBOR. This structure represents a modest reduction in risk as higher rates tend to increase the cash flow under the basis swap to the issuer. CFaR for this portfolio is $1.62

million. Notice that although the reduction in CFaR is modest, there is a meaningful reduction in *expected* debt service as well.

Portfolio 5 – $200mm VRBs, $100mm 68% LIBOR and BMA/LIBOR Basis swaps

Portfolio 5 is really just the combination of Portfolios 3 and 4. One might expect to just add the CFaR from each of those portfolios to get a CFaR for Portfolio 5 of $2.01 million. Actually, CFaR for Portfolio 5 is about $100,000 less at $1.91million. The reason for this apparent inconsistency cuts to the heart of how calculating risk can often be anything but intuitive. Depending upon the correlation between any two portfolios and without going into detail, in risk terms, one plus one can equal anything between zero and two, but not 3!

Portfolio 6 – Portfolio 5 plus $20 mm cash

Portfolio 6 takes Portfolio 5 and adds $20 million of cash. Even at this relatively low level of portfolio complexity, it becomes difficult to have a strong intuitive feel for how much risk reduction $20 million of cash might contribute to the mix. CFaR for Portfolio 6 is $1.58 million reducing risk from Portfolio 5 by $336,000 or about 17.6%.

Table 1 shows a summary of these CFaR results for each portfolio. Unless otherwise noted, numbers in this table are in $ millions.

Table 1 – CFaR Summary

Portfolio	Description (amounts are millions)	CFaR	% to Port 1
1	100 VRB	$1,709,000	-
2	100 VRB plus 20 cash	1,242,000	-27.3%
3	100 VRB plus 68% LIBOR swap	390,000	-77.2%
4	100 VRB plus 100 Basis swap	1,624,000	-4.9%
5	200 VRB plus 100 LIBOR and 100 Basis swaps	1,915,000	12.0%
6	200 VRB, 100 LIBOR and Basis swaps, 20 cash	1,578,000	-7.6%

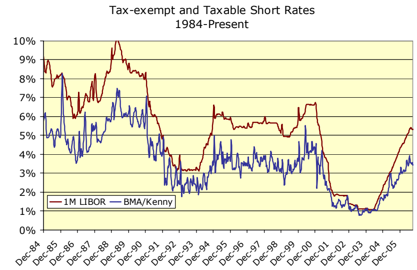
How CFaR Changes with Markets

Of course markets and expectations about them will evolve and change over time. As a risk metric that moves with markets, CFaR can immediately capture the essence of new market expectations. In this section, we first make changes to the interest rate model for LIBOR and examine the effects those changes have on CFaR. We next cover changes to the BMA/LIBOR ratio model. In both of these sections, changes are made to three model parameters: the starting rate, the average rate, and the rate volatility.

Higher Initial LIBOR Rate

Imagine an environment where inflation has presented itself and the Fed has tapped on the monetary brakes a few times in response. Current LIBOR is near 8%. What impact does this have on CFaR? Although there is still some academic debate on this topic, higher rates have empirically tended to lead to higher absolute rate volatility. That is, a 1%

change in short term rates over a given time period is more likely to happen when rate levels are at 10% then when they at 1%. The interest rate model used in this paper captures this feature of interest rate behavior and as such, the absolute dollar level of CFaR will tend to rise with higher rates. In fact, CFaR in Portfolio 1, the simple 20 year variable rate issue, goes up by 43% to $2.45 million. As expected, Portfolio 3, the synthetic fixed rate bonds using 68% LIBOR swap, is least effected given the risk and CFaR in that structure is more driven by the BMA/LIBOR ratio variability, not pure rate volatility.



Expectations about average rates are often significantly influenced by the then current market environment.

Higher Average LIBOR Rate

If economic conditions have changed such that it appears that long term rates are more likely over a more distant horizon, increasing the average LIBOR rate in the interest rate model would be prudent. If the average rate in the LIBOR rate model is increased from 6% to 8%, for the same reason described above, absolute volatility and risk as reflected in CFaR will also rise. Generally, CFaRs across all portfolios increase by nearly 30%.

Higher Rate Volatility

The volatility input into the rate model in some ways can be interpreted as our level of conviction about the future: greater uncertainty about the future leads to a higher volatility parameter. In the extreme example, if one knew exactly what interest rates would be in the future, an interest rate model could still be used; the volatility input would simply be set to 0. This would simply give the expected (and perfectly accurate!) cash flows for the portfolio in question. If this were the case, CFaR would also be 0 just like CFaR for purely fixed rate bonds.

In our base case model, LIBOR rate volatility is 25%. If we increase this to 30%, reflecting greater market uncertainty, CFaR in Portfolio 1 increases by 23% to $2.1 million. CFaR in the synthetic fixed structure in Portfolio 3 is essentially unaffected, again because it is the BMA/LIBOR ratio driving risk in that structure. The impact on CFaR in the other portfolios falls between these two extremes but in general is related to that particular portfolio's CFaR sensitivity to interest rates.

Table 2 below summarizes the impact on CFaR for each of the six portfolios resulting from changes to the LIBOR interest rate model inputs described above.

Table 2
Rate Model Effects on CFaR

Port #	Original CFaR	**From:** Initial LIB= 4% **To:** Initial LIB= 8%	% Change	Avg LIB= 6% Avg LIB= 8%	% Change	LIB Vol= 25% LIB Vol= 30%	% Change
1	$1,709,000	$2,450,000	43.4%	$2,187,000	28.0%	$2,110,000	23.5%
2	1,242,000	1,738,000	39.9%	1,595,000	28.4%	1,493,000	20.2%
3	390,000	477,000	22.3%	499,000	27.9%	391,000	0.3%
4	1,624,000	2,206,000	35.8%	2,111,000	30.0%	1,904,000	17.2%
5	1,915,000	2,495,000	30.3%	2,474,000	29.2%	2,156,000	12.6%
6	1,578,000	2,029,000	28.6%	2,049,000	29.8%	1,729,000	9.6%

The next few paragraphs discuss the impact on CFaR resulting from changes to the inputs in the BMA/LIBOR ratio model.

Higher Initial BMA/LIBOR
If, for technical market reasons, or if the rate market returns to an environment similar to the one in the early 2000s, the initial BMA/LIBOR ratio model might require adjustment. We assume that these conditions exist and as a result they warrant an increase in the initial BMA/LIBOR ratio from 68% to 78%. What happens to CFaR for our six portfolios?

This change has negligible effect on the CFaR of the VRBs in Portfolio 1. However, the basis risk present in the synthetic fixed rate structure in Portfolio 3 leads to a significant increase in CFaR from $390,000 to $431,000 or a little over 10%.

The changes in CFaR from the other portfolios reflect the degree of basis risk present in them. For instance, as reflected by CFaR in absolute terms Portfolio 6 is less risky than Portfolio 5 due to the additional hedging effects of $20 million cash. However, the impact of changing the ratio model has a greater effect on Portfolio 6 then Portfolio 5.

Higher Average BMA/LIBOR
If expectations change about long term average BMA/LIBOR ratios, most likely because of a significant revision to current tax law, the analyst should consider an adjustment to the average BMA/LIBOR ratio in the model. For this adjustment, we change the long term BMA/LIBOR average from 68% to 72%.

Obviously, this will have an impact on expected cost as now BMA will perform "closer" to LIBOR as the value of tax-exemption has decreased, but how will CFaR change. The portfolios that contain the $100 million basis swap are affected most with CFaR in those portfolios increasing by 10% to 13%. CFaR in the other portfolios also increased but by smaller increments, 5% to 7%.

Similar to the change in rate volatility described above, an increase in BMA/LIBOR ratio volatility from 6% to 8% has the greatest percentage impact on the portfolio whose risk is most driven by this basis relationship, the synthetic fixed rate bonds in Portfolio 3. In this case, the CFaR in Portfolio 3 increases by over 35% to $529,000.

Table 3 summarizes the BMA/LIBOR ratio model's effect on CFaR.

Table 3
BMA/LIBOR Ratio Model Effects on CFaR

Port #	Original CFaR	**From:** Init Ratio= 68% **To:** Init Ratio= 78%	% Change	Avg Ratio= 68% Avg Ratio= 72%	% Change	Ratio Vol = 6% Ratio Vol = 8%	% Change
1	$1,709,000	1,742,000	1.9%	1,798,000	5.2%	1,737,000	1.6%
2	1,242,000	1,268,000	2.1%	1,328,000	6.9%	1,292,000	4.0%
3	390,000	431,000	10.5%	418,000	7.2%	529,000	35.6%
4	1,624,000	1,717,000	5.7%	1,795,000	10.5%	1,804,000	11.1%
5	1,915,000	2,049,000	7.0%	2,123,000	10.9%	2,233,000	16.6%
6	1,578,000	1,694,000	7.4%	1,797,000	13.9%	1,930,000	22.3%

In summary and as expected, the rate model changes tended to impact the purely "rate" sensitive portfolio the most. Portfolio 1 holds this distinction as it is comprised entirely of variable rate bonds. The least interest rate sensitive portfolio is Portfolio 3 as its rate risk has been hedged with a 68% LIBOR swap. The rate impact on CFaR for the remaining portfolios fell between these two boundaries.

In the case of changing the BMA/LIBOR ratio model parameters, the roles of Portfolio 1 and 3 reversed, though they remained the boundaries. Portfolio 1 was least sensitive to ratio model changes and Portfolio 3 with its basis exposure was affected the most.

The Risk Intersection – Corner of Market and Credit

The implications for using CFaR not only in discussions of market risk but also as an addition to the credit lexicon are far reaching. For instance, one frequently deployed measure in tax-exempt finance is maximum annual debt service (MADS). However, the actual calculation of MADS rarely lives up to its own superlative name. MADS is almost always calculated based upon one or more static and heroic assumptions about the expected long term average for short term rates: BMA, LIBOR, and the relationship between the two. Put this way, MADS seems to fall short of being the "maximum" that it ought to be. However, one can calculate traditional MADS, add CFaR to it, and arrive at something that's worthy of the acronym, a MADS complete with a confidence interval of one's choosing, 95% MADS for instance.

High quality decisions that comprehensively balance expected risk versus benefit, particularly in dynamic markets, are impossible to make without adequate tools.

Following this line of reasoning, coverage ratios with confidence intervals can be derived in the same way they always have, dividing forecasted revenues by debt service. However, a greater degree of analytic meaning has entered into the debt service denominator; the potential market impact on coverage ratios is appropriately captured.

Summary

"The path is smooth that leadeth on to danger."
William Shakespeare

Traditional measures of risk, including fixed/floating mix, are ill-suited to assess comprehensively the complex composition of modern tax-exempt capital structures. They are insensitive to market dynamics, and unable to appropriately handle hybrid exposure such as basis risk or capped floating structures. Further, as current ratios they offer little if any information about risk on a more distant horizon.

This article offers a definition, calculation methodology, and graphical explanation of Cash Flow at Risk (CFaR), a powerful alternative to traditional measures offering significant advantages: most notably, a comprehensive framework for handling complex exposure profiles and immediate sensitivity to market dynamics. Using CFaR, we examine six specific capital structure examples and compare them, first using static model inputs, then in relation to changing market conditions and model assumptions. CFaR analysis can lead to more comprehensive understanding of tactical risk taking, improved measurement of aggregate capital market exposure, and a more clear sense of strategic direction.

With tools now broadly available to calculate CFaR, issuers, investors, rating agencies, and governing bodies can use CFaR to better evaluate risk and make more informed capital market decisions.

Photos courtesy of the New York Public Library Digital Gallery.

For information on employing these models to analyze CFaR for your portfolio, please contact sales@intuitive-analytics.com for a list of Intuitive Analytics partners.